CS 383
Exam 2 Solutions

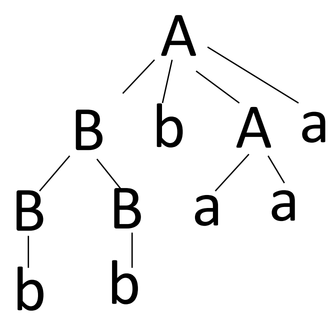
You only need to answer the first 5 questions. #6 is a bonus question to give you something to do if you finish early. Questions (1) through (5) are worth 20 points each.

1. Here is a list of languages. Which of these are context-free? You don't need to give a justification for your answer, so you can guess randomly and have a 1/32 chance of getting them all right…

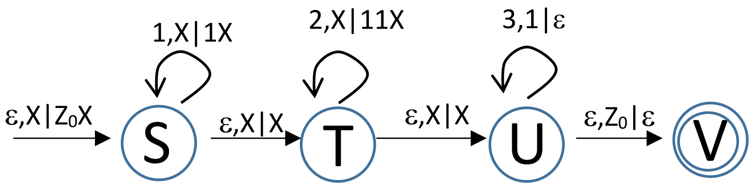
   a. Strings of 0s, 1s, and 2s with more 1s than 0s and more 2s than 1s
   b. Strings of 0s and 1s with odd length that have a 1 as the center digit. For example, 00100, 11111, and 11101 are all in this language.
   c. $\{0^n1^n0^m1^m \mid n,m >= 0\}$
   d. $\{0^n1^m0^m1^n \mid n,m >= 0\}$
   e. $\{0^n1^m0^n1^m \mid n,m >= 0 : \}$

   (a) is not CF: $z = 0^p1^{p+1}2^{p+2}$ is not pumpable within ths language
   (b) is CF: Make a PDA with 2 states. In the first state push X whenever you see a 0 or 1. Nondeterminitically on a 1 go to the second state that pops an X whenever it sees a 0 or 1.
   (c) is CF: it is L_1L_2 where L_1 and L_2 are both the CF language $\{0^n1^n\}$
   (d) is CF: Push 0s then push 1s, then on 0s pop 1 and finally on 1s pop 0s.
   (e) is not CF. The string $0^p1^p0^p1^p$ is not pumpable.

2. Here is a grammar. Draw a parse tree for the derivation of bbbaaa
   A => BbAa | aa
   B => BB | b | ε



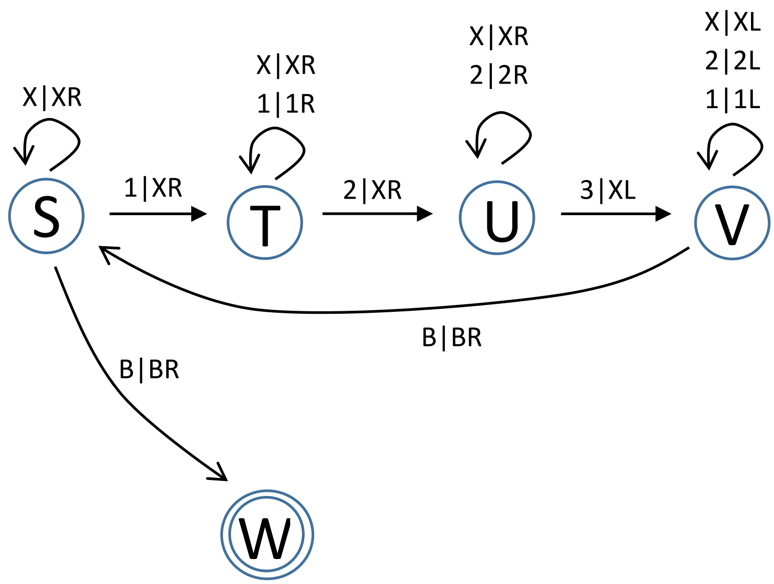
3. Construct a PDA that accepts by final state the language $\{1^n 2^m 3^{n+2m} \mid n, m >= 0\}$



4. Give a careful proof that the language $\{1^n 2^m 3^{n*m} \mid n,m >= 0\}$ is not context-free.

   Suppose this language is context-free; let p be its pumping constant. Consider the string $z = 1^p 2^p 3^{p*p}$. Let z = uvwxy be any pumping decomposition of z, with $|vwx| <= p$. vwx can't contain both 1s and 3s. Changing 1s or 2s without changing 3s won't keep us in the language; neither will changing 3s without changing 1s or 2s. The only possibility is for vwx to contain both 2s and 3. Note that to be in our language, if there are p 1s each 2 must correspond to p 3s. If we pump once: uv^2wx^2y has p 1s, some additional 2s and fewer than p additional 3s.Since it does not have p 3s for every 2, it is not in the language. Our string z is thus not pumpable, so the language is not context-free.

5. Construct a Turing Machine that accepts the language $\{1^n 2^n 3^n \mid n > 0\}$



Here is the idea. In one pass from S to V we overwrite a 1, a 2, and a 3 with Xs. In node V we go back to the very beginning of the string and return to node S. If in node S we find that the entire string has been overwritten with Xs we transition to node W and accept the input.